

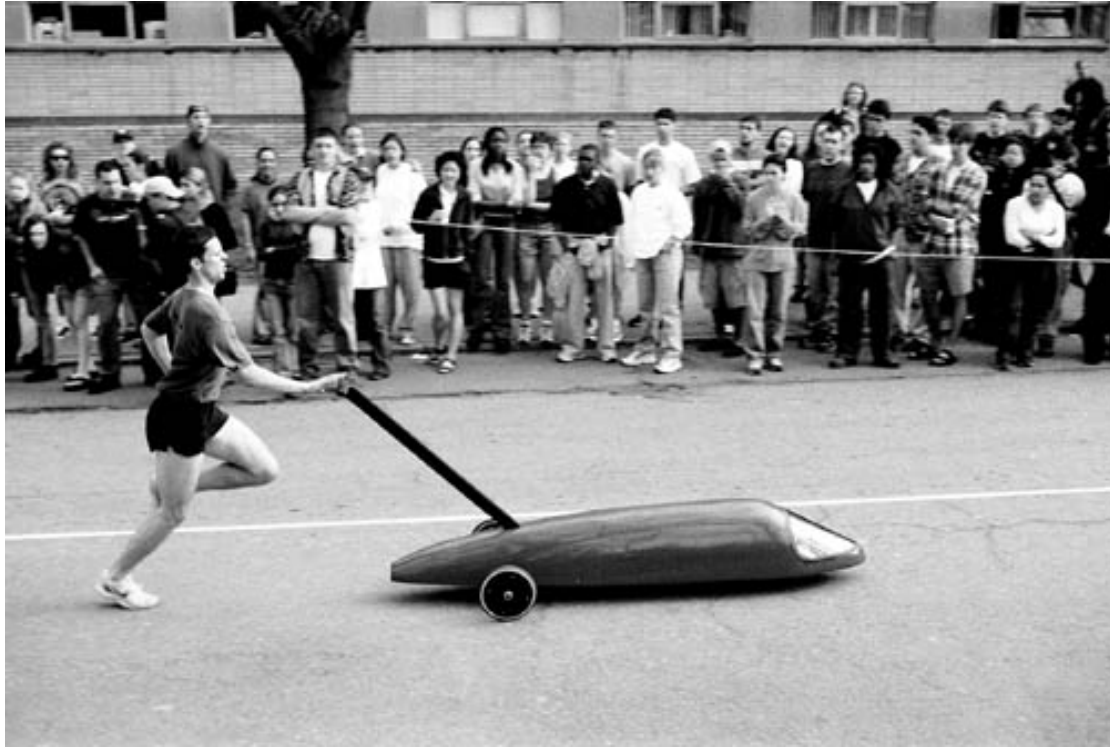
R2-D2 GOES TO BUGGY



Emily Yeh & Anastassia Kornilova



BUGGY



VEHICLE SAFETY IN THE REAL WORLD



R2-D2 enters THE races



R2D2 Goes to Buggy by Anastassia Kornilova & Emily Yeh

BASICS OF THE MODEL

Track: Helix with fixed width and varying parameters

Parameters:

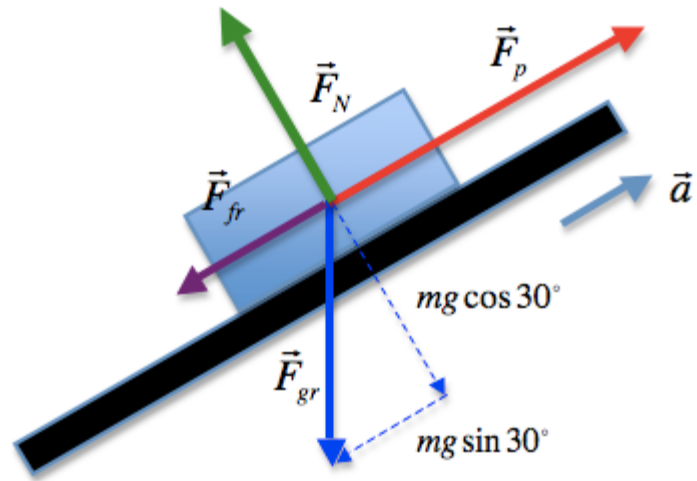
Model 1. Varying helix radii

Model 2. Varying helix slope

Buggy Control (with R2D2): acceleration

Buggy Evolution: circular motion

PHYSICS: CIRCLES and INCLINES



$$\sum F_x = F_p - F_{fr} - mg \sin 30^\circ = ma$$

$$\sum F_y = F_N - mg \cos 30^\circ = 0$$

$$F_{fr} = \mu F_N$$

$$F_N = mg \cos 30^\circ$$

Figure from: <http://physatwes.com/SecondLawHonors.aspx>

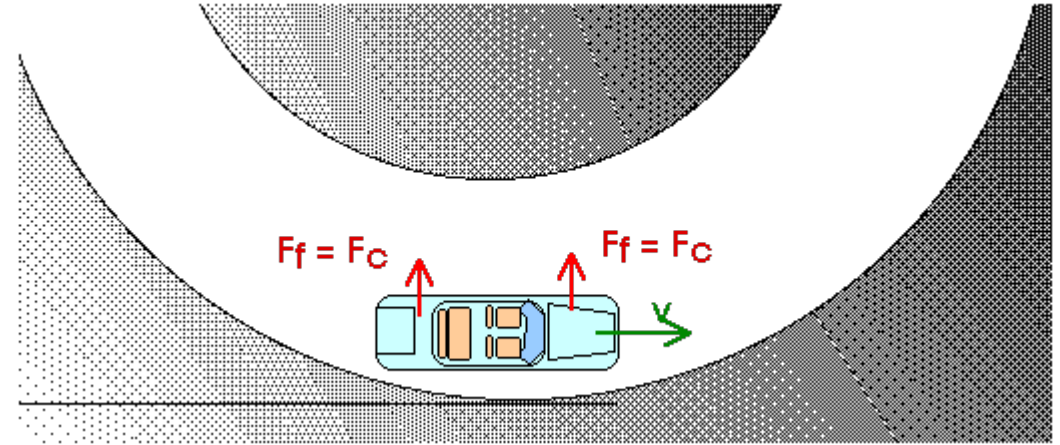
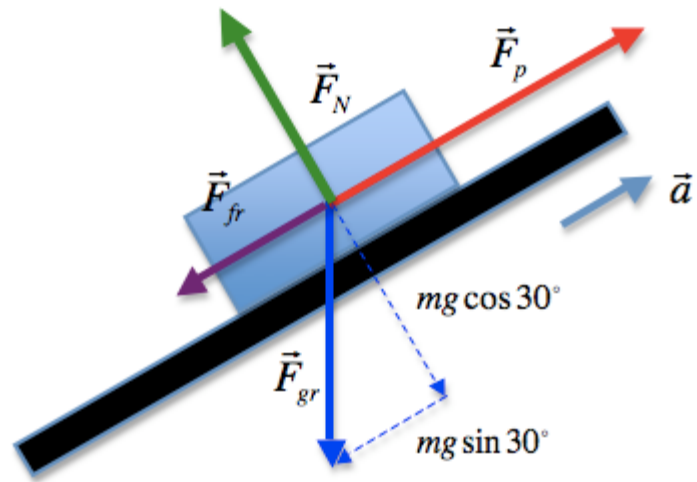


Figure from: <http://www.ux1.eiu.edu/~cfadd/1150/05UCMGrav/Curve.html>

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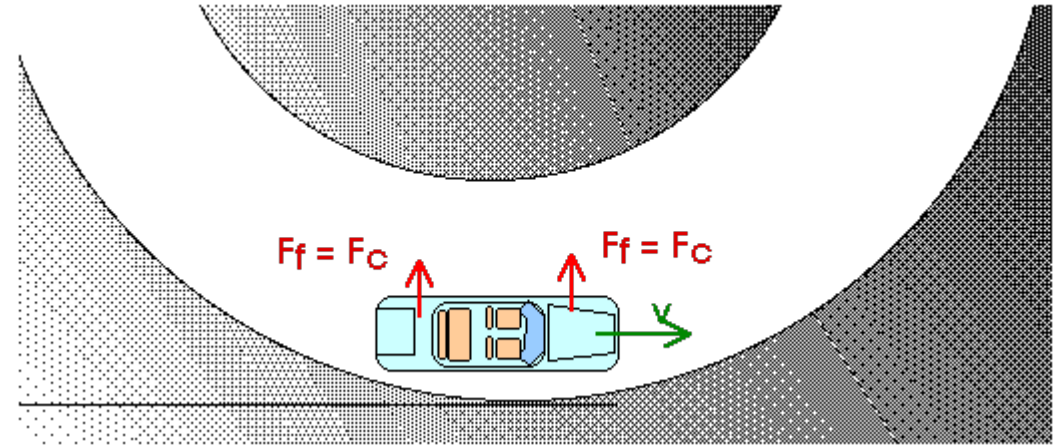


Figure from: <http://www.ux1.eiu.edu/~cfadd/1150/05UCMGrav/Curve.html>

$$\mu * m * g * \cos(\theta) = m * v^2 / r$$

$$v^2 / fr = r$$

Are we safe? Are we efficient?

- Buggy's Radius: $\text{buggyR} = v^2 / \mu r$
- Stay in track: $\text{trackR} \leq \text{buggyR} \leq \text{trackR} + \text{width}$
- Maintain reasonable velocity: $v_{\text{Min}} \leq v \leq v_{\text{Max}}$

APProach to PROblem

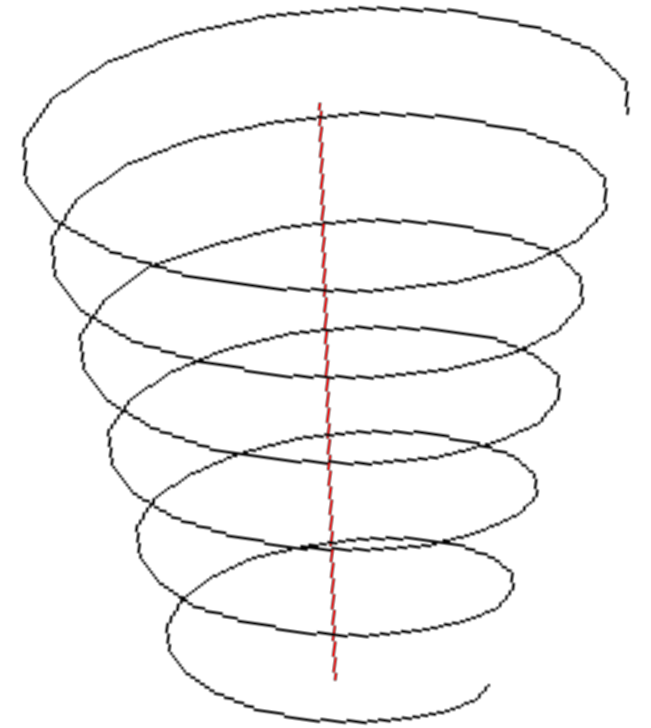
```
initial conditions ->
( /* track generation decision */
  tRate = -inRate OR tRate = -outRate OR tRate=0

  /* acceleration */
  a := A if safe OR
  a := -B if safe OR
  a := 0 if safe

/* ODEs - continuous evolution by physics */
)@loop invariant
/* ensure final safety conditions */
```

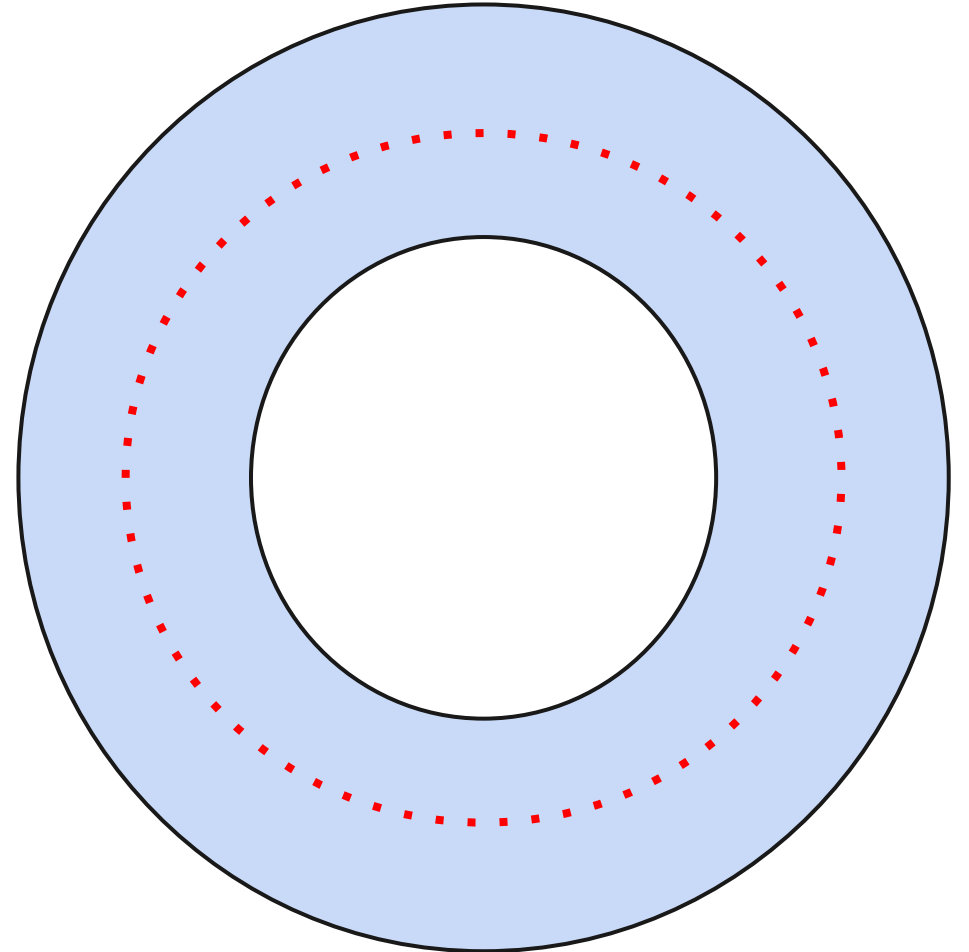
MODEL 1. THE HELIX

- Constant slope
- Radius can expand, shrink or remain the same
- Two challenges:
 - How to test for safety?
 - How to ensure safe condition exists?

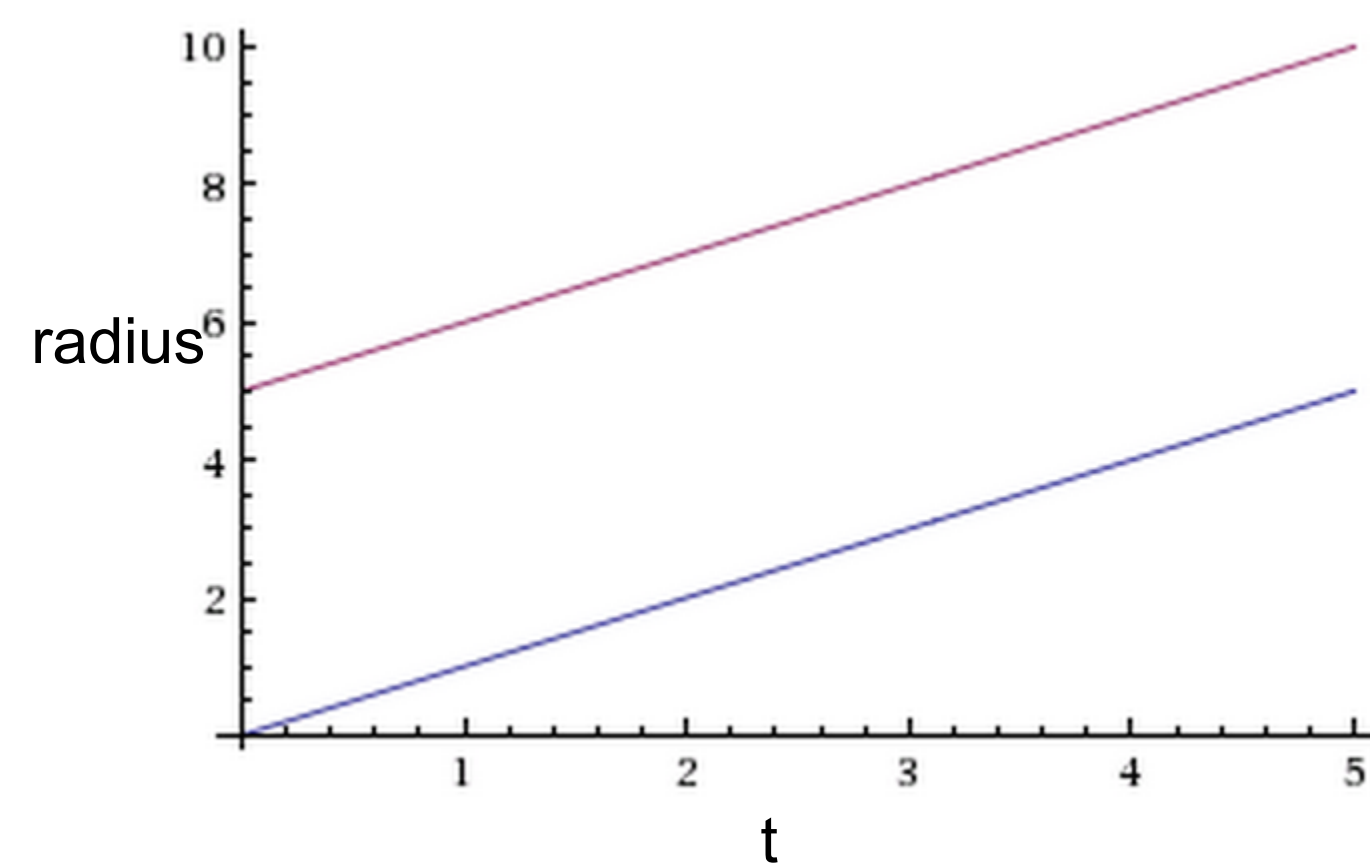


SIMPLE case: THE circular Track

- Track Radius does not change
- Can coast safely ($a=0$)

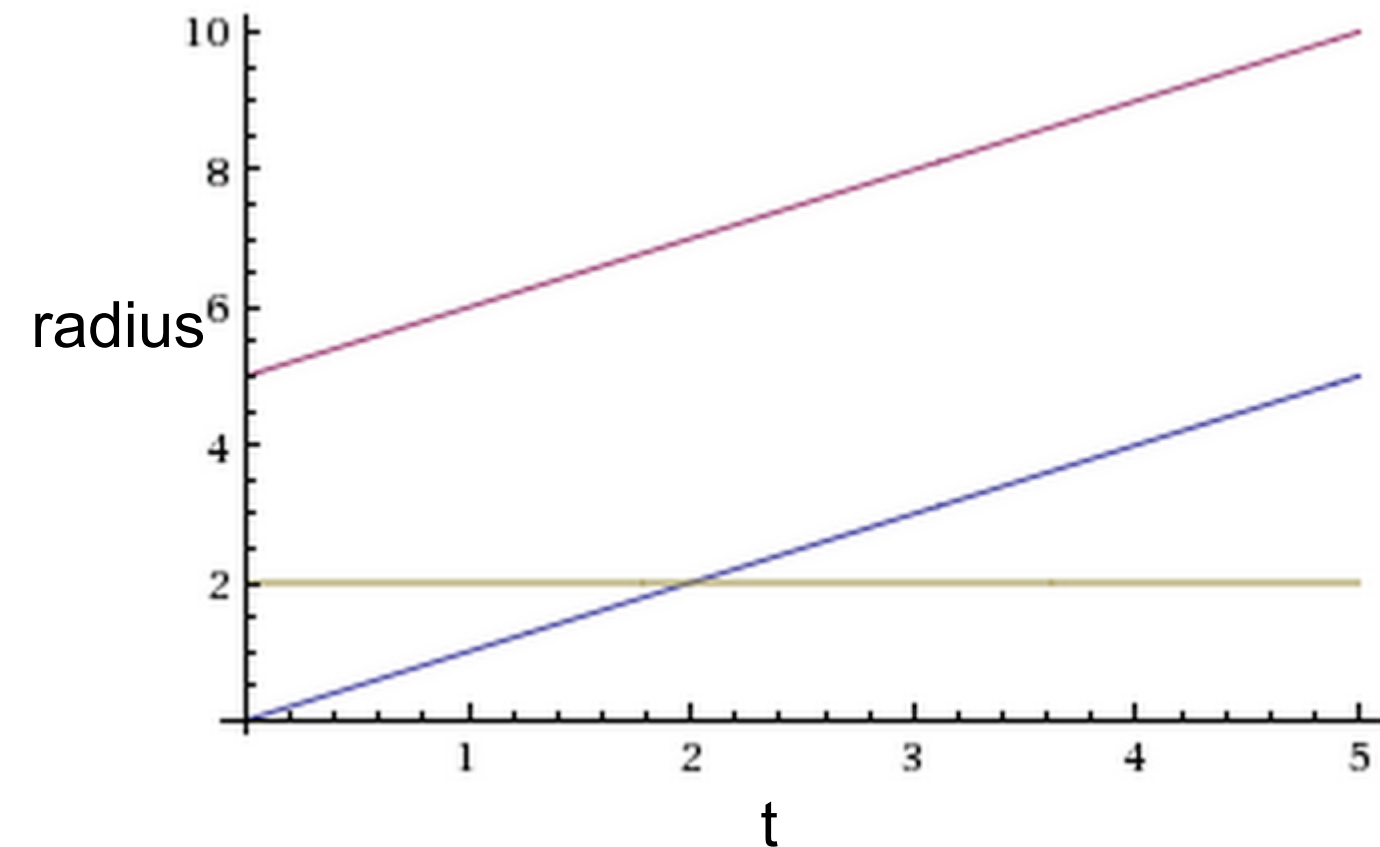


FINDING a safe decision



$$\begin{aligned}\text{inner} &= \text{trackR_1} + \text{outRate} * t \\ \text{outer} &= \text{trackR_1} + \text{width} + \text{outRate} * t\end{aligned}$$

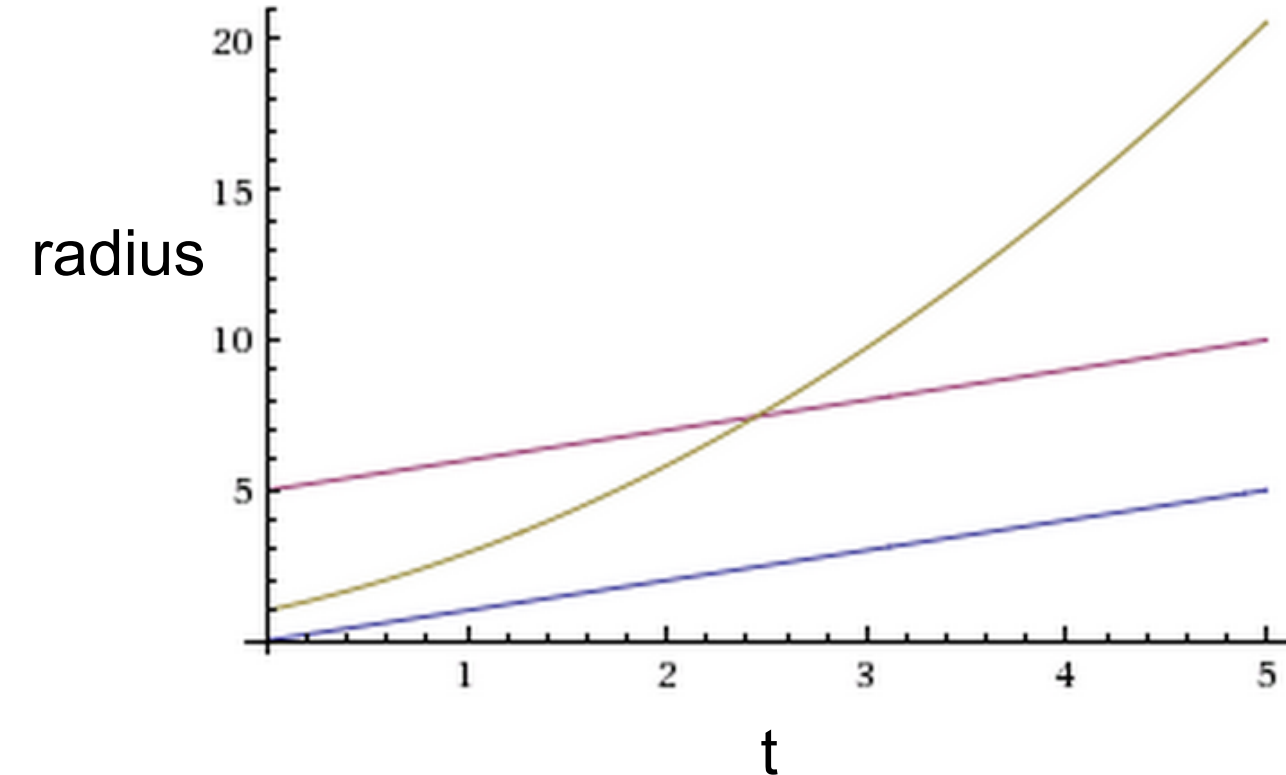
FINDING a safe decision: COASTING?



$$\begin{aligned}\text{inner} &= \text{trackR_1} + \text{outRate} * t \\ \text{outer} &= \text{trackR_1} + \text{width} + \text{outRate} * t\end{aligned}$$

FINDING a safe DECISION: acceLerating?

$$\text{buggyR} = (v + A * t)^2 / fr$$



FINDING a safe DECISION: DEFINING CONSTRAINTS

- When the track is expanding:
 - Coast safely in the outer half
 - Accelerate safely in the inner one
- When the track is shrinking:
 - Coast safely in the inner half
 - Brake in the outer half
- When track is not changing:
 - Coast safely everywhere
- Define formulas to ensure these

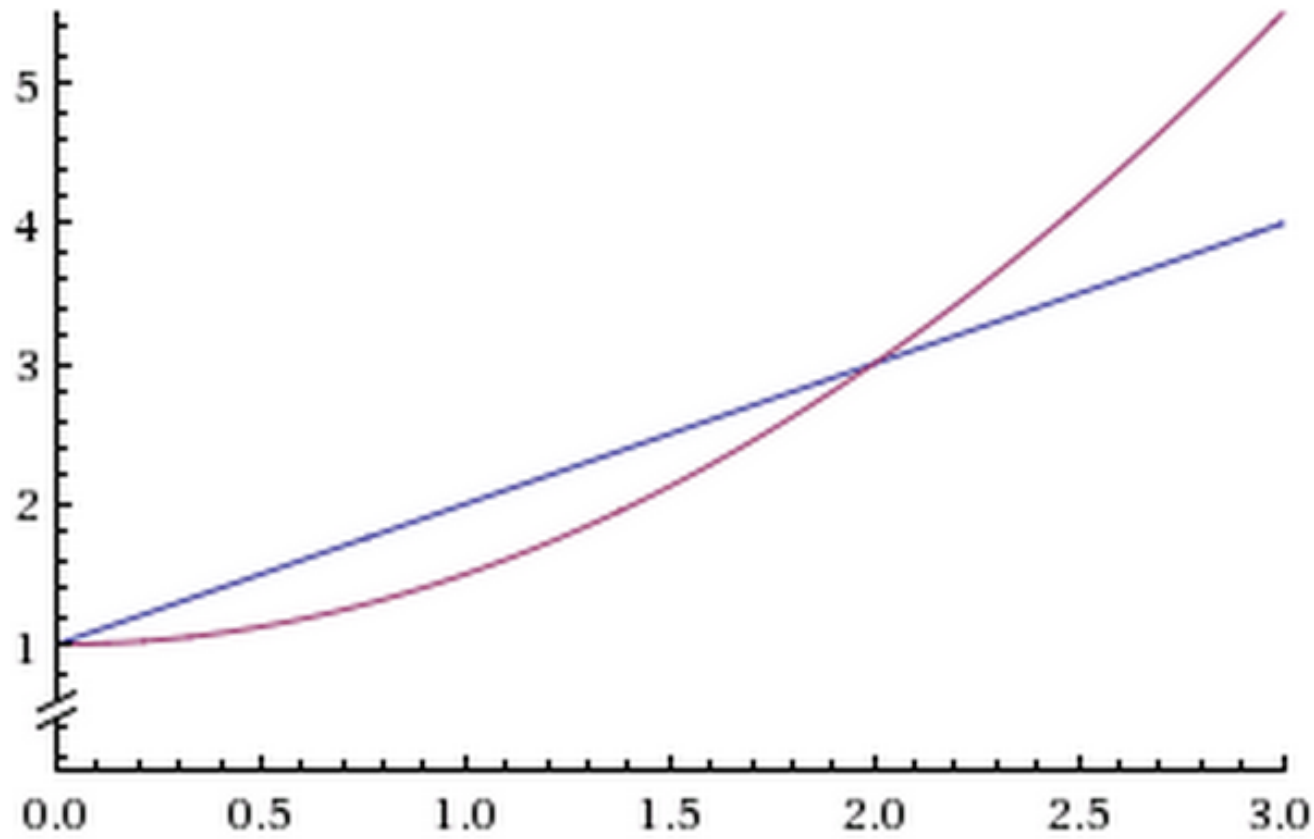
constraint: coasting on expanding Track

- Outer edge is moving away - can't cause collision
- Inner Edge will approach middle the faster

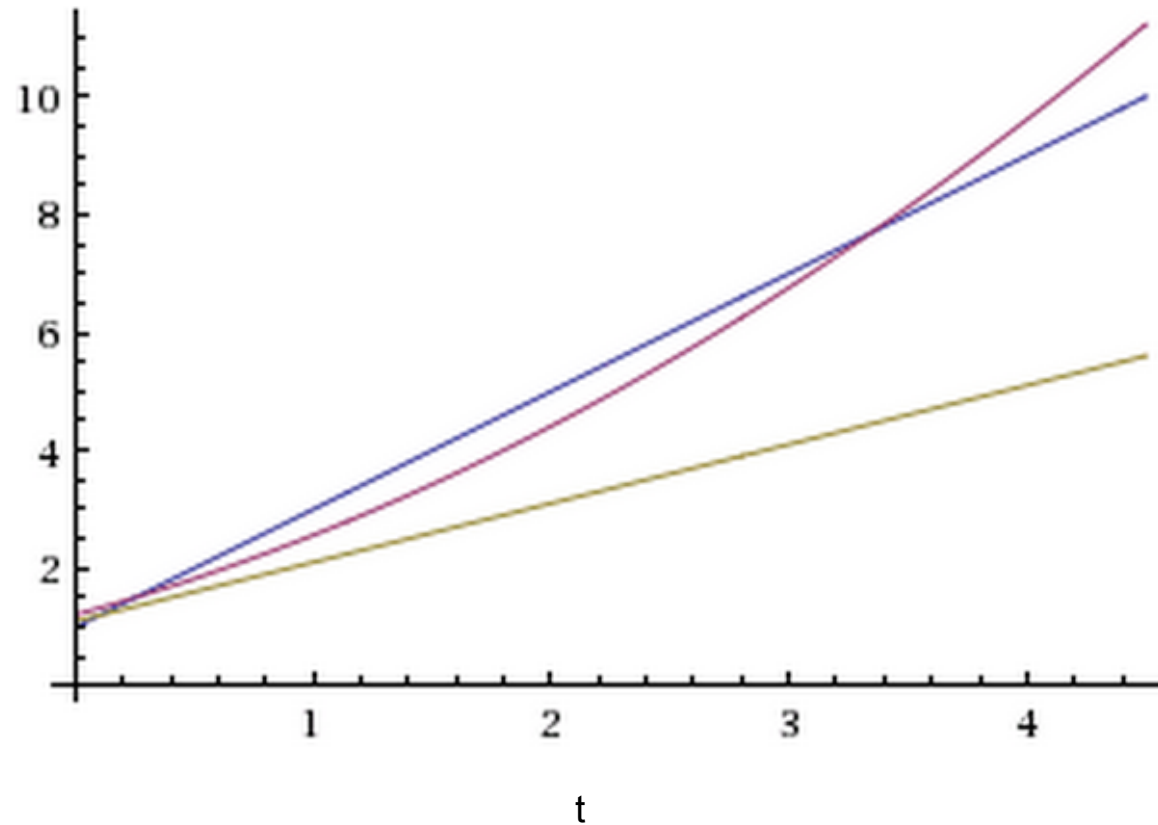
$$trackR + width/2 \geq trackR + inRate * t \quad \forall 0 \leq t \leq T$$

$$width/2 \geq inRate * T$$

ANOTHER CHALLENGE FOR CONSTRAINTS: safe at THE end, BUT NOT THE MIDDLE!



ANOTHER CHALLENGE FOR CONSTRAINTS: SAFE AT THE END, BUT NOT THE MIDDLE!



constraint: accelerating on EXPANDING TRACK

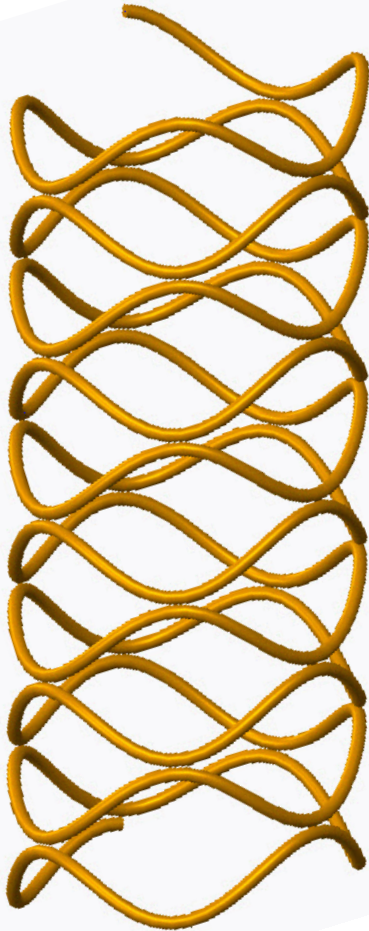
Outer edge: $2 * vMax * A * T / fr + A^2 * T^2 / fr \leq width/2 + outRate * T$

Inner Edge: $outRate \leq 2 * vMin * A / fr$

SAFETY OF HELIX MODEL

- Define 6 constraints for guaranteed safe decisions
- Constraints use constants and remain true
- Inequalities use extreme values - can be extended

MODEL 2. THE HELIX WITH HILLS

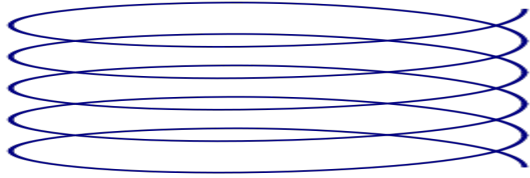


Track **slope** changes over time.
Track **radius** stays the same.

Safety: Don't Crash!

*Track chooses new slope change,
not new slope.*

case 1. FLAT SLOPE



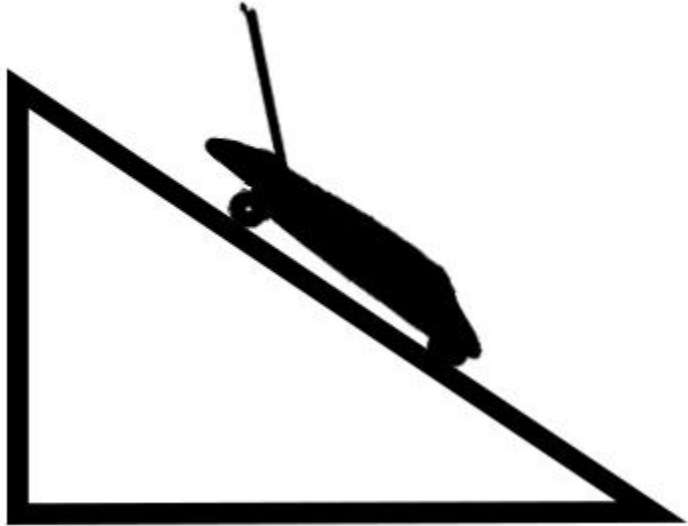
- Slope = 0
- $F_g = G \cdot \cos(0) = G$

Risks:

- No new risks introduced by slope

case 2. DOWNHILL

- $F_g > 0$



Risks:

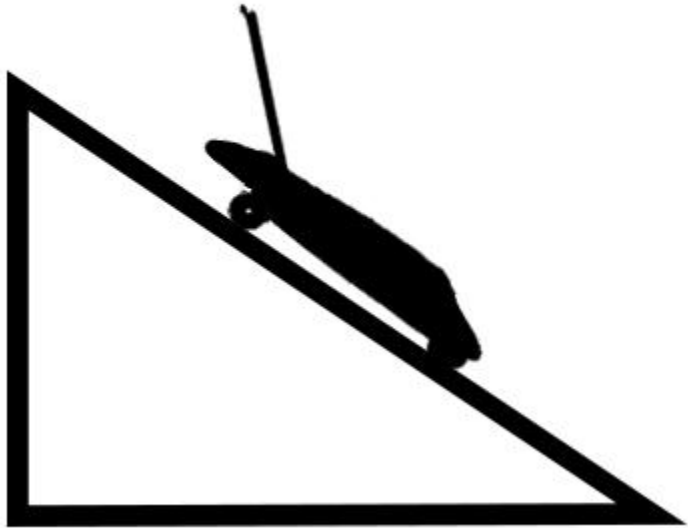
- Crashing into the outer edge (high v)

DOWNHILL safety

Constraints added to ensure:

Inner half of track means...
acceleration is safe.

Outer half of track means...
deceleration is safe.



DOWNHILL safety

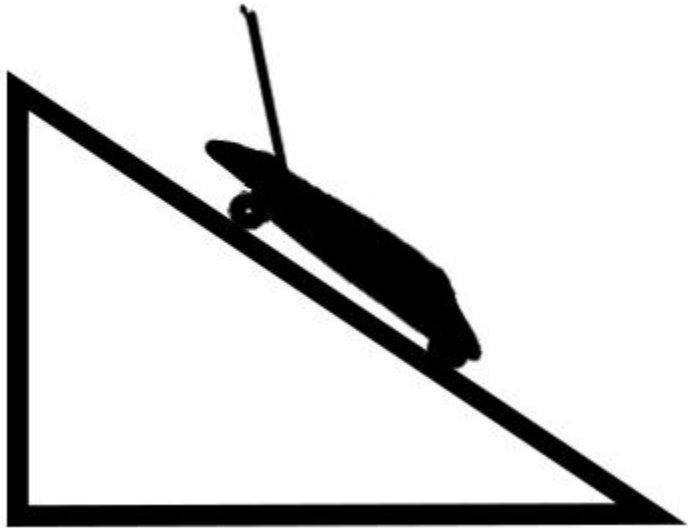
Constraints added to ensure:

Inner half of track means...
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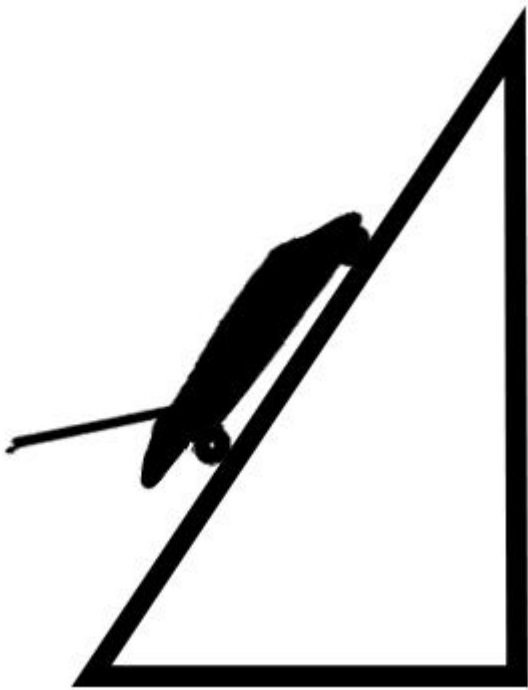
$$vMax \geq (vMax + vMin)/2 + (A + G) * T + (T^2)/2$$

Outer half of track means...
deceleration is safe.

$$vMax \geq vMax + (-B + G) * T + (T^2)/2$$



CASE 3. UPHILL



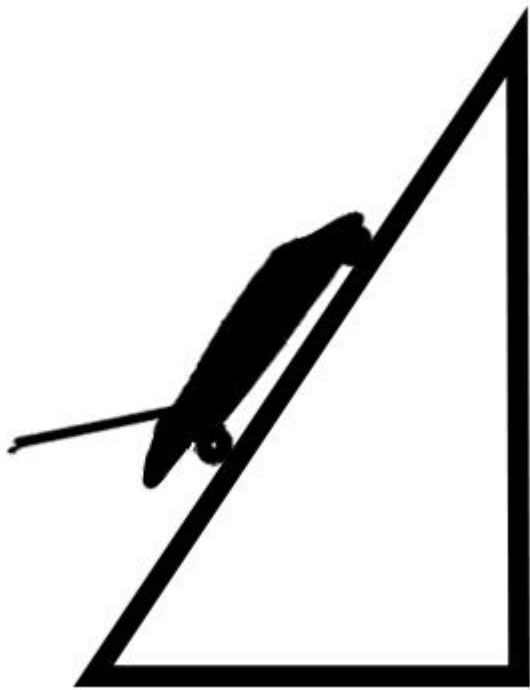
- $F_g < 0$

Risks:

- Crashing into inner edge (low v)

UPHILL safety

Constraints added to ensure:

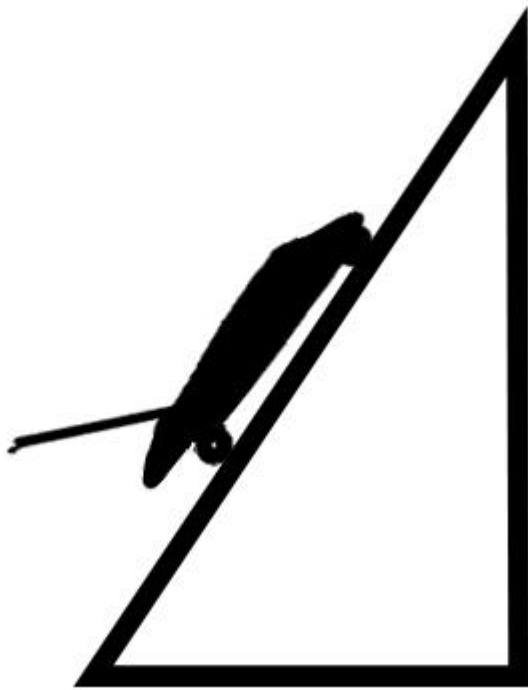


Inner half of track means...
acceleration is safe.

Outer half of track means...
deceleration is safe.

UPHILL safety

Constraints added to ensure:



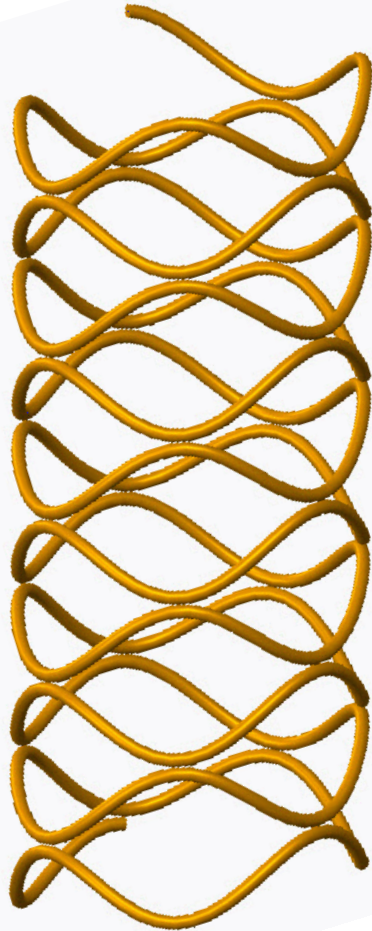
Inner half of track means...
acceleration is safe.

$$vMin \leq vMin + (A - G) * T - (T^2)/2$$

Outer half of track means...
deceleration is safe.

$$vMin \leq (vMin + vMax)/2 + (-B - G) * T - (T^2)/2$$

MODEL CONTROLS

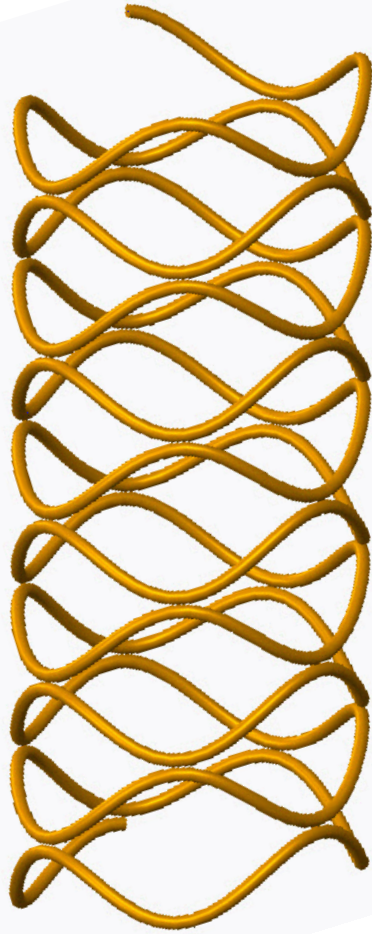


- Choices: A, -B, 0
- Makes choice based on tests that make sure we won't crash

Tests:

- Crash into inner edge?
- Crash into outer edge?

MODEL 2 summary

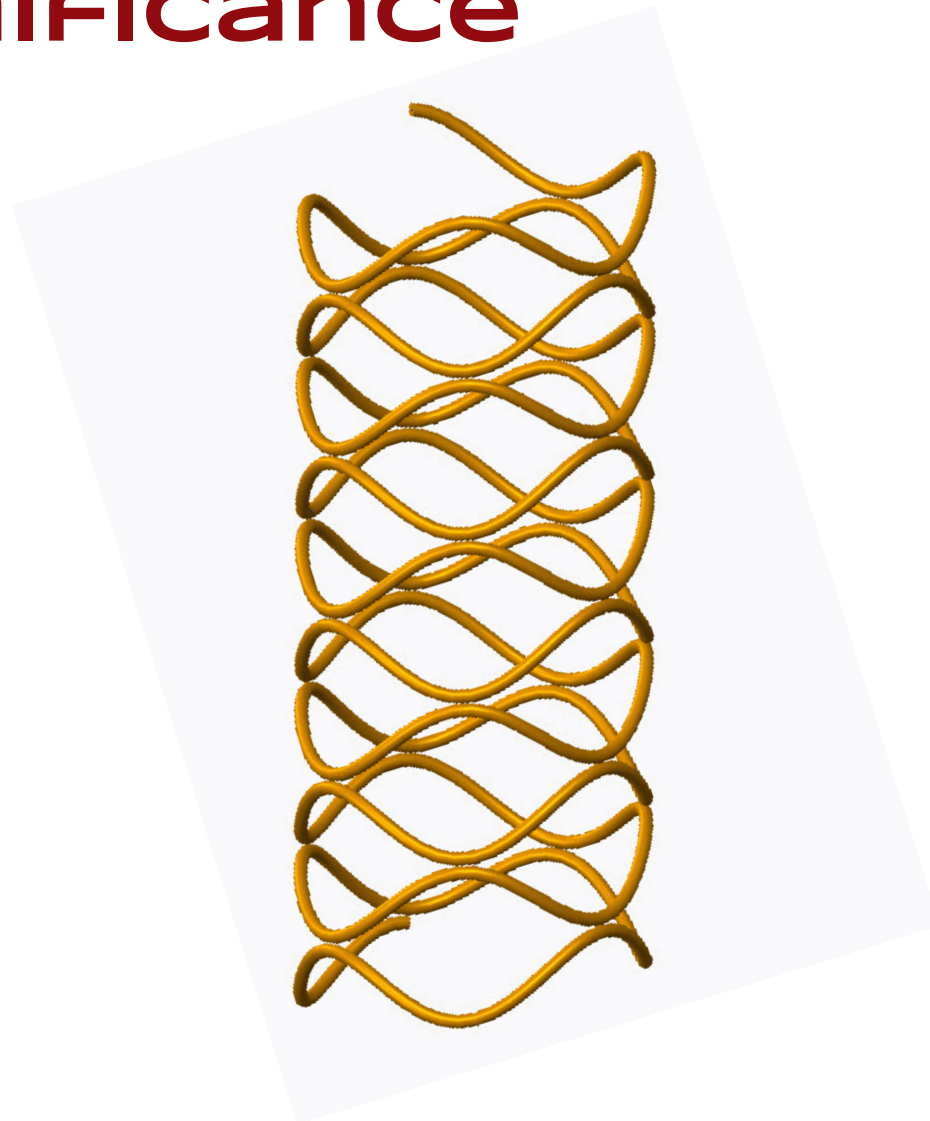
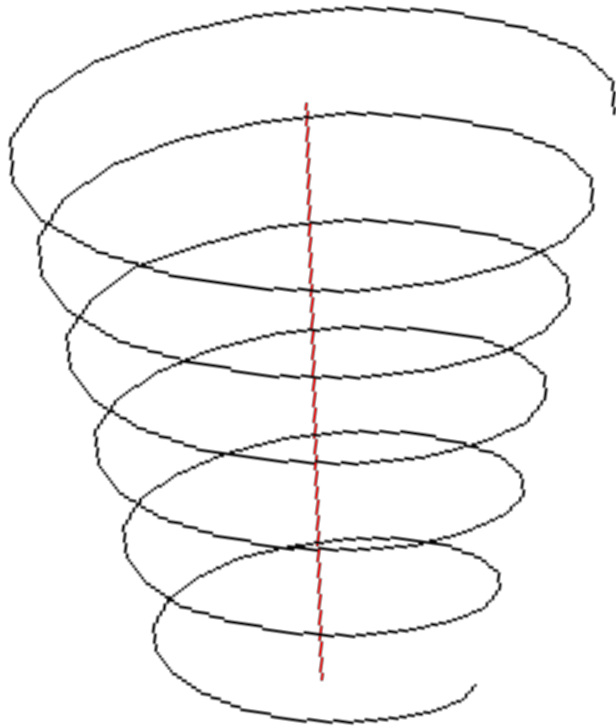


- Makes decisions based on upcoming slope changes
- Constraints use constants
- Constraints and tests ensure safety
- Model is limited by conservative constraints
- Model doesn't analyze F_{fr} change

FUTURE DIRECTIONS

- Better efficiency: find best path around track segment
- More Diverse Tracks
 - Combine changes in slope and radius
 - Allow for straight segments
- Looking ahead in tracks to find better paths
- Less synchronized controls

summary & significance



THANK you.