Infinitely Long Road, One Lane, Many Cars

The previous version was very simple, and resultantly, the chicken had a very simple strategy. However, in the presence of many (possibly infinitely many) cars, the same strategy may not work. However, with the proper restrictions on the cars, we can guarantee that the chicken can make non-zero progress towards the other side of the road. Furthermore, the proof of the one car version is very instructive, in that we only need to be slightly modified its proof to generalize for the case of many cars.

Firstly, consider a simplified road with just one lane (so in this context, each car's y-position is the same). If there is a constant stream of traffic with no gap between the cars, of course, the chicken cannot make progress. Hence, we must place a simple restriction that there is some space between the cars. Let us formalize this.

We use the same notation as before, but extend that for the cars as follows. We introduce a sort C for cars. For each car $i \in C$, let $\mathbf{c}(i)$ denote the position of i, and u_x denote the x-velocity of i. For this simplified version, we will also have that the width w, length l and height h of every car is the same. Further, to express that fact that the cars are all in one lane, we will have $c_y(i) = c_y(j) \wedge u_x = u_x(j)$ for every $i \neq j \in C$. For simplicity, we will just write u_x for the velocity of any one car. Finally, there is some gap between the cars, which we express by $c_x(i) - c_x(j) < -l \vee c_x(i) - c_x(j) > l$ for cars $i \neq j \in C$.

Now let us build the quantified hybrid program for this game.

$$A \equiv k_x = 0 \wedge k_y = 0 \wedge k_z = 0 \wedge v_x = 0 \wedge v_y = 0 \wedge v_z = 0$$

$$B \equiv \forall i : C.\ 0 < c_y(i) - \frac{w}{2} \wedge c_y(i) + \frac{w}{2} < H \wedge c_z(i) = 0 \wedge u_x > 0$$

$$C \equiv \forall i : C.\ (\forall j : C.\ i \neq j \rightarrow (c_y(i) = c_y(j) \wedge (c_x(i) - c_x(j) < -l \vee c_x(i) - c_x(j) > l)))$$

$$D \equiv H > 0 \wedge w > 0 \wedge h \geq 0 \wedge l \geq 0 \wedge T > 0$$

$$\alpha \equiv k_z := h + 1; v_x := 0; v_y := 0; v_z := 0$$

$$\beta \equiv k_z := 0; v_x := *; v_y := *; v_z := 0; ?(v_y > 0); \gamma_1; \gamma_2; \textsf{if } v_x = u_x \textsf{ then } ?(\forall i : C.\ k_x < c_x(i) - l \vee c_x(i) < k_x \vee t_1(i) > T) \textsf{ else } \gamma_3; \gamma_4; \delta_1; \delta_2 \textsf{ fi}$$

$$\gamma_1 \equiv \forall i : C.\ t_1(i) := \frac{c_y(i) - \frac{w}{2} - k_y}{v_y}$$

$$\gamma_2 \equiv \forall i : C.\ t_2(i) := \frac{c_y(i) + \frac{w}{2} - k_y}{v_y}$$

$$\gamma_3 \equiv \forall i : C.\ s_1(i) := \frac{c_x(i) - k_x}{v_x - u_x}$$

$$\gamma_4 \equiv \forall i : C.\ s_2(i) := \frac{c_x(i) - l - k_x}{v_x - u_x}$$

$$\delta_1 \equiv ?\,(\forall i : C.\ s_1(i) \leq s_2(i) \rightarrow ((t_2(i) < s_1(i) \vee s_2(i) < t_1(i) \vee t_1(i) > T \vee s_1(i) > T))$$

$$\delta_2 \equiv ?\,(\forall i : C.\ s_1(i) > s_2(i) \rightarrow (t_2(i) < s_2(i) \vee s_1(i) < t_1(i) \vee t_1(i) > T \vee s_2(i) > T))$$

$$\epsilon \equiv \forall i : C.\ \{c_x(i)' = u_x, k_x' = v_x, k_y' = v_y, k_z' = v_z, t' = 1, t \leq T\}$$

$$\chi \equiv t := 0; (\alpha \cup \beta); \epsilon$$

Then we wish to prove the following:

$$A \wedge B \to \langle \chi^* \rangle\, k_y \geq H$$

We will prove this in multiple steps:

Let $p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})$ be the predicate $\langle \chi^* \rangle\, k_y \geq H$, where the vectors in the arguments have their obvious meanings. Here, the set $\{i \in C : \mathbf{c}(i)\}$ is taken to represent the vector/list of all the positions of all the cars, indexed by $i \in C$.

$$\langle * \rangle,\ \forall,\ \mathit{MP}\ \dfrac{\mathit{US}\ \dfrac{\langle ; \rangle,\ \langle := \rangle,\ \langle \cup \rangle\ \dfrac{\cdots\ \dfrac{Y_1}{\forall \mathbf{k}, \{i \in C : \mathbf{c}(i)\}\,(k_y \geq H \vee X_1 \vee X_2 \to p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})) \to (A \wedge B \wedge C \wedge D \to p(\mathbf{k}, \mathbf{c}))}}{\forall \mathbf{k}, \{i \in C : \mathbf{c}(i)\}\,((k_y \geq H \vee \langle \chi \rangle\, p(\mathbf{k}, \mathbf{c})) \to p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})) \to (A \wedge B \wedge C \wedge D \to p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\}))}}{\forall \mathbf{k}, \{i \in C : \mathbf{c}(i)\}\,((k_y \geq H \vee \langle \chi \rangle\, \langle \chi^* \rangle\, k_y \geq H) \to \langle \chi^* \rangle\, k_y \geq H) \to (A \wedge B \wedge C \wedge D \to \langle \chi^* \rangle\, k_y \geq H)}}{A \wedge B \wedge C \wedge D \to \langle \chi^* \rangle\, k_y \geq H}$$

where

$$X_1 \equiv \{t := 0\}\, \langle \alpha \rangle\, \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})$$
$$X_2 \equiv \{t := 0\}\, \langle \beta \rangle\, \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})$$

Reduction of X_1:

Let

$$\zeta_1 \equiv \forall i : C.\ c_x(i) := c_x(i) + u_x \cdot \tilde{r}; k_x := k_x + v_x \cdot \tilde{r}; k_y := k_y + v_y \cdot \tilde{r}; k_z := k_z + v_z \cdot \tilde{r}; t := t + \tilde{r}$$
$$\zeta_2 \equiv \forall i : C.\ c_x(i) := c_x(i) + u_x \cdot r; k_x := k_x + v_x \cdot r; k_y := k_y + v_y \cdot r; k_z := k_z + v_z \cdot r; t := t + r$$

$$\langle ; \rangle,\ \langle := \rangle\ \dfrac{\langle ' \rangle\ \dfrac{\mathit{subst}\ \dfrac{\mathbb{R}\ \dfrac{\exists 0 \leq r \leq T.\ p((k_x, k_y, h+1), \{i \in C : (c_x(i) + u_x \cdot r, c_y(i), c_z(i))\})}{\exists r \geq 0\,((\forall 0 \leq \tilde{r} \leq r\ \tilde{r} \leq T) \wedge p((k_x, k_y, h+1), \{i \in C : (c_x(i) + u_x \cdot r, c_y(i), c_z(i))\}))}}{\{t := 0; k_z := h+1; v_x := 0; v_y := 0; v_z := 0\} \exists r \geq 0\,((\forall 0 \leq \tilde{r} \leq r\, \langle \zeta_1 \rangle\, t \leq T) \wedge \langle \zeta_2 \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\}))}}{\{t := 0; k_z := h+1; v_x := 0; v_y := 0; v_z := 0\}\, \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})}}{\{t := 0\}\, \langle \alpha \rangle\, \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})}$$

Reduction of X_2:

Let

$$\zeta_3 \equiv t := 0; k_z := 0; v_x := \widetilde{v_x}; v_y := \widetilde{v_y}; v_z := 0$$

$$\zeta_4 \equiv t := 0; k_z := 0; v_x := \widetilde{v_x}; v_y := \widetilde{v_y}; v_z := 0; \forall i : C.\ t_1(i) := \frac{c_y - \frac{w}{2} - k_y}{\widetilde{v_y}}; \forall i : C.\ t_2(i) := \frac{c_y + \frac{w}{2} - k_y}{\widetilde{v_y}}$$

$$\zeta_5 \equiv ?(\forall i : C.\ k_x < c_x(i) - l \lor c_x(i) < k_x \lor t_1(i) > T)$$

$$\langle ; \rangle,\ \langle := \rangle,\ \langle :* \rangle \dfrac{\langle ? \rangle,\ \langle := \rangle,\ sub \dfrac{if,\ sub,\ \exists\lor \dfrac{Z_1 \lor Z_2}{\exists \widetilde{v_x}, \widetilde{v_y}.\ \widetilde{v_y} > 0 \land \{\zeta_4\} \langle \textsf{if } v_x = u_x \textsf{ then } \zeta_5 \textsf{ else } \gamma_3; \gamma_4; \delta_1; \delta_2 \textsf{ fi} \rangle \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\}))}}{\exists \widetilde{v_x}, \widetilde{v_y}.\ \{\zeta_3\} \langle ?(v_y > 0) \rangle \langle \gamma_1 \rangle \langle \gamma_2 \rangle \langle \textsf{if } v_x = u_x \textsf{ then } \zeta_5 \textsf{ else } \gamma_3; \gamma_4; \delta_1; \delta_2 \textsf{ fi} \rangle \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\}))}}{\{t := 0\} \langle \beta \rangle \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\}))}$$

where $\widetilde{v_x}$ and $\widetilde{v_y}$ are fresh, and where

$$Z_1 \equiv \exists \widetilde{v_x}, \widetilde{v_y}.\ \widetilde{v_y} > 0 \land \widetilde{v_x} = u_x \land \{\zeta_4\} \langle \zeta_5 \rangle \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})$$
$$Z_2 \equiv \exists \widetilde{v_x}, \widetilde{v_y}.\ \widetilde{v_y} > 0 \land \widetilde{v_x} \neq u_x \land \{\zeta_4\} \langle \gamma_3; \gamma_4; \delta_1; \delta_2 \rangle \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})$$

Reduction of Z_1:

Let

$$F_1 \equiv \widetilde{v_y} > 0 \land \widetilde{v_x} = u_x \land \forall i : C.\ \left(k_x < c_x(i) - l \lor c_x(i) < k_x \lor \frac{c_y(i) - \frac{w}{2} - k_y}{\widetilde{v_y}} > T\right)$$

$$\langle ? \rangle,\ \langle := \rangle,\ sub \dfrac{\langle ' \rangle,\ sub,\ \mathbb{R} \dfrac{ax \dfrac{\exists \widetilde{v_x}, \widetilde{v_y}, 0 \le r \le T.\ (F_1 \land p((k_x + \widetilde{v_x} \cdot r, k_y + \widetilde{v_y} \cdot r, 0), \{i \in C : c_x(i) + u_x \cdot r, c_y(i), c_z(i)\}))}{\exists \widetilde{v_x}, \widetilde{v_y}.\ (F_1 \land \exists 0 \le r \le T.\ p((k_x + \widetilde{v_x} \cdot r, k_y + \widetilde{v_y} \cdot r, 0), \{i \in C : c_x(i) + u_x \cdot r, c_y(i), c_z(i)\}))}}{\exists \widetilde{v_x}, \widetilde{v_y}.\ \widetilde{v_y} > 0 \land \widetilde{v_x} = u_x \land \forall i : C.\ \left(k_x < c_x(i) - l \lor c_x(i) < k_x \lor \frac{c_y(i) - \frac{w}{2} - k_y}{\widetilde{v_y}} > T\right) \land \{\zeta_4\} \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})}}{\exists \widetilde{v_x}, \widetilde{v_y}.\ \widetilde{v_y} > 0 \land \widetilde{v_x} = u_x \land \{\zeta_4\} \langle \zeta_5 \rangle \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})}$$

Reduction of Z_2:

Let

$$\zeta_6 \equiv t := 0; k_z := 0; v_x := \widetilde{v_x}; v_y := \widetilde{v_y}; v_z := 0; t_1(i) := \frac{c_y(i) - \frac{w}{2} - k_y}{\widetilde{v_y}}; t_2(i) := \frac{c_y(i) + \frac{w}{2} - k_y}{\widetilde{v_y}}; s_1(i) := \frac{c_x(i) - k_x}{\widetilde{v_x} - u_x}; s_2(i) := \frac{c_x(i) - l - k_x}{\widetilde{v_x} - u_x}$$

$$\widetilde{t_1}(i) \equiv \frac{c_y(i) - \frac{w}{2} - k_y}{\widetilde{v_y}}$$

$$\widetilde{t_2}(i) \equiv \frac{c_y(i) + \frac{w}{2} - k_y}{\widetilde{v_y}}$$

$$\widetilde{s_1}(i) \equiv \frac{c_x(i) - k_x}{\widetilde{v_x} - u_x}$$

$$\widetilde{s_2}(i) \equiv \frac{c_x(i) - l - k_x}{\widetilde{v_x} - u_x}$$

$$\langle ; \rangle,\ \langle := \rangle,\ sub\ \dfrac{\langle ? \rangle,\ \rightarrow,\ sub\ \dfrac{\langle ' \rangle,\ sub,\ \mathbb{R},\ \exists\wedge\ \dfrac{Z_3}{\exists \widetilde{v_x}, \widetilde{v_y}.\ \widetilde{v_y} > 0 \wedge \widetilde{v_x} \neq u_x \wedge G_1 \wedge G_2 \wedge \{\zeta_6\}\, \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})}}{\exists \widetilde{v_x}, \widetilde{v_y}.\ \widetilde{v_y} > 0 \wedge \widetilde{v_x} \neq u_x \wedge \{\zeta_6\}\, \langle \delta_1 \rangle\, \langle \delta_2 \rangle\, \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})}}{\exists \widetilde{v_x}, \widetilde{v_y}.\ \widetilde{v_y} > 0 \wedge \widetilde{v_x} \neq u_x \wedge \{\zeta_4\}\, \langle \gamma_3; \gamma_4; \delta_1; \delta_2 \rangle\, \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})}$$

where

$$Z_3 \equiv \exists \widetilde{v_x}, \widetilde{v_y}, 0 \le r \le T.\ \widetilde{v_y} > 0 \wedge \widetilde{v_x} \neq u_x \wedge G_1 \wedge G_2 \wedge p((k_x + \widetilde{v_x} \cdot r, k_y + \widetilde{v_y} \cdot r, 0), \{i \in C : (c_x(i) + u_x \cdot r, c_y, c_z)\})$$
$$G_1 \equiv \forall i : C.\ \widetilde{s_1}(i) \le \widetilde{s_2}(i) \rightarrow \left(\widetilde{t_2}(i) < \widetilde{s_1}(i) \vee \widetilde{s_2}(i) < \widetilde{t_1}(i) \vee \widetilde{t_1}(i) > T \vee \widetilde{s_1}(i) > T\right)$$
$$G_2 \equiv \forall i : C.\ \widetilde{s_1}(i) > \widetilde{s_2}(i) \rightarrow \left(\widetilde{t_2}(i) < \widetilde{s_2}(i) \vee \widetilde{s_1}(i) < \widetilde{t_1}(i) \vee \widetilde{t_1}(i) > T \vee \widetilde{s_2}(i) > T\right)$$

Proof of Y_1:

Let

$$W_1 \equiv \exists 0 \le r \le T.\ p((k_x, k_y, h+1), \{i \in C : (c_x(i) + u_x \cdot r, c_y(i), c_z(i))\})$$
$$W_2 \equiv \exists \widetilde{v_x}, \widetilde{v_y}, 0 \le r \le T.\ \left(F_1 \wedge p((k_x + \widetilde{v_x} \cdot r, k_y + \widetilde{v_y} \cdot r, 0), \{i \in C : c_x(i) + u_x \cdot r, c_y(i), c_z(i))\})\right)$$
$$W_3 \equiv \exists \widetilde{v_x}, \widetilde{v_y}, 0 \le r \le T.\ \widetilde{v_y} > 0 \wedge \widetilde{v_x} \neq u_x \wedge G_1 \wedge G_2 \wedge p((k_x + \widetilde{v_x} \cdot r, k_y + \widetilde{v_y} \cdot r, 0), \{i \in C : (c_x(i) + u_x \cdot r, c_y, c_z)\})$$
$$E \equiv A \wedge B \wedge C \wedge D$$

Now, we can continue on with the first proof:

$$\dfrac{\rightarrow\ \dfrac{\forall \mathbf{k}, \{i \in C : \mathbf{c}(i)\}\, ((k_y \ge H \vee W_1 \vee W_2 \vee W_3) \rightarrow p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})) \rightarrow (\neg(E) \vee (E \wedge p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})))}{\forall \mathbf{k}, \{i \in C : \mathbf{c}(i)\}\, ((k_y \ge H \vee W_1 \vee W_2 \vee W_3) \rightarrow p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})) \rightarrow (E \rightarrow p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\}))}}{\forall \mathbf{k}, \{i \in C : \mathbf{c}(i)\}\, ((k_y \ge H \vee \{t := 0\}\, \langle \alpha \rangle\, \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\}) \vee \{t := 0\}\, \langle \beta \rangle\, \langle \epsilon \rangle\, p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})) \rightarrow p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})) \rightarrow (E \rightarrow p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\}))}$$

As a reminder, we have

$$
\begin{aligned}
A &\equiv k_x = 0 \wedge k_y = 0 \wedge k_z = 0 \wedge v_x = 0 \wedge v_y = 0 \wedge v_z = 0 \\
B &\equiv \forall i : C.\ 0 < c_y(i) - \frac{w}{2} \wedge c_y(i) + \frac{w}{2} < H \wedge c_z(i) = 0 \wedge u_x > 0 \\
C &\equiv \forall i : C.\ (\forall j : C.\ i \neq j \rightarrow (c_y(i) = c_y(j) \wedge (c_x(i) - c_x(j) < -l \vee c_x(i) - c_x(j) > l))) \\
D &\equiv H > 0 \wedge w > 0 \wedge h \geq 0 \wedge l \geq 0 \wedge T > 0 \\
F_1 &\equiv \widetilde{v_y} > 0 \wedge \widetilde{v_x} = u_x \wedge \forall i : C.\ \left(k_x < c_x(i) - l \vee c_x(i) < k_x \vee \frac{c_y(i) - \frac{w}{2} - k_y}{\widetilde{v_y}} > T \right)
\end{aligned}
$$

If the left hand side $\forall \mathbf{k}, \{i \in C : \mathbf{c}(i)\} \left((k_y \geq H \vee W_1 \vee W_2 \vee W_3 \vee W_4) \rightarrow p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})\right)$ implies $\neg E$, then there is nothing to show, so we may instead simply prove

$$E \wedge \forall \mathbf{k}, \{i \in C : \mathbf{c}(i)\} \left((k_y \geq H \vee W_1 \vee W_2 \vee W_3 \vee W_4) \rightarrow p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})\right) \rightarrow p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})$$

Then the proof continues as:

$$eq\ \frac{E \wedge \forall \mathbf{k}, \{i \in C : \mathbf{c}(i)\} \left((k_y \geq H \vee W_1 \vee W_2 \vee W_3 \vee W_4) \rightarrow p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})\right) \rightarrow p((0,0,0), \{i \in C : c_x(i), c_y(i), 0\})}{E \forall \mathbf{k}, \{i \in C : \mathbf{c}(i)\} \left((k_y \geq H \vee W_1 \vee W_2 \vee W_3 \vee W_4) \rightarrow p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})\right) \rightarrow p(\mathbf{k}, \{i \in C : \mathbf{c}(i)\})}$$

Now, we can consider the W_i's as "rules" that decide which values of $\mathbf{k}$ and $\mathbf{c}$ makes $p(\mathbf{k}, \mathbf{c})$ true, and prove efficiency by working backwards.

Rule W_1 allows the chicken to simply hover in place at a greater height; from the rule, one should note that this move does not actually help the chicken directly make progress towards the goal.

Rule W_2 allows the chicken to move at a constant non-zero velocity that it chose (such that its x-velocity matches the velocity of the cars, and such that it will not collide with any one of the cars within a period of T time; this is determined by calculating the time $t_1(i)$ that it would take for the chicken to enter the path of car i and checking that time is greater than T, and by making sure that the chicken is not travelling along side of a car, in which case the chicken is guaranteed to hit the car given enough time).

Rule W_3 allows the chicken to move at a constant non-zero velocity that it chose (such that its x-velocity is different from the velocity of the cars, and such that it will not collide with any one of the cars within a period of T time; this is done by calculating the times $t_1(i)$ and $t_2(i)$ it takes the chicken to enter and exit the path of car i and the times $s_1(i)$ and $s_2(i)$ it takes for the car i's length to coincide with the chicken's x position, and determining if the interval with endpoints $t_1(i)$ and $t_2(i)$ intersects the interval with endpoints $s_1(i)$ and $s_2(i)$.

Initially, the chicken starts at $(0,0,0)$ and each car i at $(c_x(i), c_y(i), 0)$. From C, we know that the $c_y(i)$'s are equal; let us denote their common value by c_y, so that each car starts at $(c_x(i), c_y, 0)$.

Now, one strategy the chicken can follow is this: wait (hover) until there is no car on the line $x = 0$, and then zoom across at a high velocity. This is a safe and sure way of reaching the goal (easy applications of rule W_1 and W_3).

Of course, this is again not very realistic. However, with this proof in hand, one can now easily add restrictions and modify the constraints of the chicken's velocity, without disrupting the existence of a solution for the chicken to cross the road.

One can easily add a cap on the chicken's velocity. Then it is not hard to see that if the chicken ever ends up in the path of a car, but cannot cross that path quickly enough, it can choose to jump and hover until the car has passed, and then continue on its merry way. Since C guarantees that every car is separated from each other by at least some gap, it is always possible for the chicken to do this.

Again, from what we have so far, it is also not difficult to see that the same problem, except allowing for varying lengths and widths and heights of the cars (so car i has width $w(i)$, length $l(i)$, and height $h(i)$), still has a solution for the chicken, *as long as the chicken can choose the height at which it will hover.*

Infinitely Long Road, Many Lanes, Many Cars

The next step from the previous scenario is to include many lanes where cars may drive. Lanes will have a common width ω, and every car is guaranteed to fit inside a line, i.e., $w(i) < \omega$. Here, cars in different lanes will be guaranteed to be separated width-wise, i.e. if car i and j are in different lanes, then we guarantee that $|c_y(i) - c_y(j)| > \frac{w(i)+w(j)}{2}$, and that $c_y(i)$ is equal to some fixed value(s) of y that is predetermined beforehand (for example, we can have three different values to model a road with three lanes). Because the proof for this situation is so similar to the proof for the case of **Infinitely Long Road, One Lane, Many Cars**, we omit it.

Infinitely Long Road, Many Lanes, Many Cars, No Jump

Finally, a modification to the choices the chicken can make can be made so that the chicken can no longer instantaneously jump, which is more true to physics.

If we allow a sufficient gap between the cars, and have a cap on the velocities of the cars (i.e., a road speed limit), it is easy to see that the chicken will still have a way to safely reach the goal on the other side of the road. Formally, if we place a speed limit S on the cars, and the maximum velocity of the chicken is M, then by having

$$|c_x(i) - l - c_y(i)| \geq S \cdot \frac{\omega}{M},$$

then the chicken has will be able to cross a lane safely (this equation represents the fact that the time it takes for a car to travel the gap between cars is at least the time it takes the chicken to cross the width of the cars w). Furthermore, since there are gaps between cars in different lanes, the chicken can choose to squat in between cars in the between lanes (i.e., choose velocity 0).

Again, the formal proof of the program for this case is similar to the proof of **Infinitely Long Road, One Lane, Many Cars**, we omit it.